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What you'll Learn About

- How to use tangent line approximations to estimate a functions value for a given value of x

$f(x) = \frac{1}{2}x^2 - x + 2.5$

$f(1.3) = 2.045$

$\int \frac{dy}{dx} = \int x - 1$

$y = \frac{1}{2}x^2 - x + C$

$2 = \frac{1}{2} - 1 + C$

$2.5 = C$

1. Consider the differential equation $\frac{dy}{dx} = x - 1$. Let y = f(x) be the particular solution to the given differential equation with initial condition f(1) = 2. Use Euler's Method, starting at x = 1 with three steps of equal size, to approximate f(1.3). Show the work that leads to your answer.

$(1,2)$ $\frac{dy}{dx} = x - 1$ $\frac{dy}{dx} = 0$ $\boxed{y = 2 + 0(x-1)}$

Δx

$y = 2 + 0(1.1 - 1)$ $\boxed{(1.1, 2)}$ $\frac{dy}{dx} = x - 1$ $\boxed{\frac{dy}{dx} = .1}$ $y = 2 + .1(x - 1.1)$

$y = 2 + .1(1.2 - 1.1)$ $\boxed{(1.2, 2.01)}$ $\frac{dy}{dx} = x - 1$ $\boxed{\frac{dy}{dx} = .2}$ $y = 2.01 + .2(x - 1.2)$

$y = 2.01 + .2(1.3 - 1.2)$ $(1.3, 2.03)$

$\boxed{f(1.3) = 2.03}$

2. Consider the differential equation $\frac{dy}{dx} = x - 2y$. Let y = f(x) be the particular solution to the given differential equation with initial condition f(2) = 1. Use Euler's Method, starting at x = 2 with three steps of equal size, to approximate f(1.7). Show the work that leads to your answer.

$(2,1)$ $\frac{dy}{dx} = x - 2y$ $\frac{dy}{dx} = 0$ $y = 1 + 0(x - 2)$

$y = 1 + 0(1.9 - 2)$ $(1.9, 1)$ $\frac{dy}{dx} = -.1$ $y = 1 - .1(x - 1.9)$

$y = 1 - .1(1.8 - 1.9)$ $(1.8, 1.01)$ $\frac{dy}{dx} = 1.8 - 2(1.01) = -.22$ $y = 1.01 - .22(x - 1.8)$

$y = 1.01 - .22(1.7 - 1.8)$ $(1.7, 1.032)$

$y = 1.01 + .022$

$\boxed{f(1.7) = 1.032}$

$\frac{dy}{dx} = x - 2y$

$dy = (x - 2y)dx$

Stuck

What you'll Learn About

- How use slopes to create a solution to a differential equation

Given the function $y = \frac{1}{2}x^2$. Find what $\boxed{\frac{dy}{dx} = x}$

At each grid point representing integers, calculate the value of the derivative and draw a short line segment with that slope.

$x = 1 \quad \frac{dy}{dx} = 1$

$x = 2 \quad \frac{dy}{dx} = 2$

$y = \frac{1}{2}x^2 - 2$

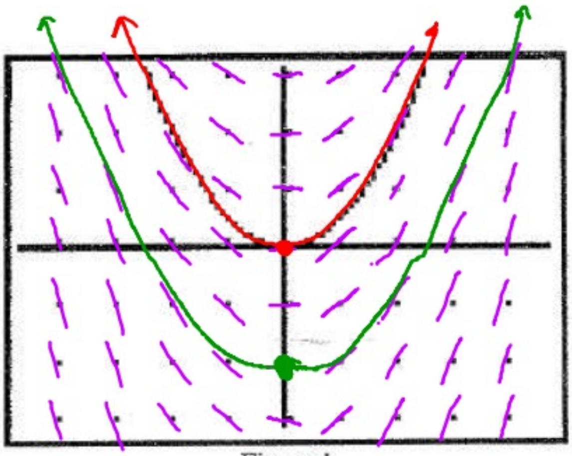


Figure 1

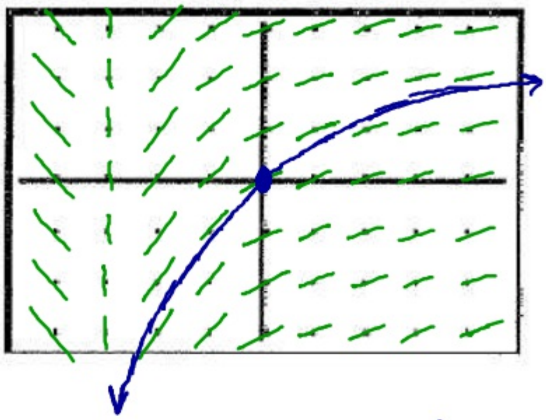
A) What family of functions seems to match all the slope fields?

quadratic

B) What is an initial condition of the function graphed?

(0,0)

4. If $\frac{dy}{dx} = \frac{1}{x+3}$, sketch the slope field



$x = 0 \quad \frac{dy}{dx} = \frac{1}{3}$

$x = 1 \quad \frac{dy}{dx} = \frac{1}{4}$

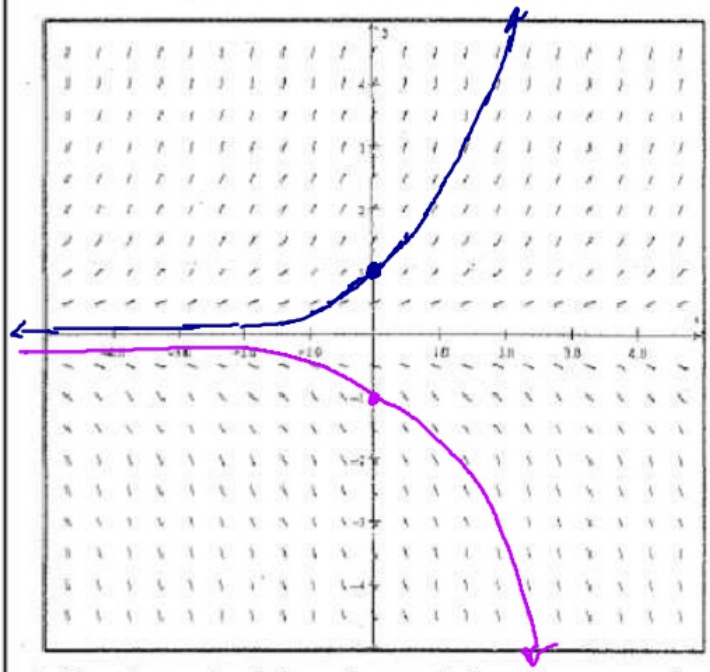
$x = -3 \quad \frac{dy}{dx} = \frac{1}{0}$

$\frac{dy}{dx} = \frac{1}{x+3}$ $\qquad (0,0)$

$\int dy = \int \frac{1}{x+3}\, dx$

$y = \ln|x+3| + C$

Given the slope field shown below answer the following questions.



a) Sketch a path of the unique solution that passes through (0, 1).

b) Sketch a path of the unique solution that passes through (0, -1).

c) What familiar functions do these resemble?

exponential

d) Given $\frac{dy}{dx} = y$, verify your guess analytically.

$$\frac{dy}{dx} = y$$

$$\int \frac{1}{y}\,dy = \int 1\,dx$$

$$\ln y = x + C$$

$$y = e^{x+C}$$

$$\boxed{y = Ce^{x}}$$

$(0,1)$ $\frac{dy}{dx} = 1$

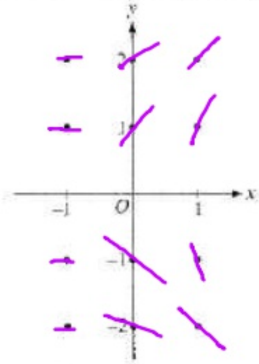
$(0,2)$ $\frac{dy}{dx} = \frac{1}{2}$

$(0,-1)$ $\frac{dy}{dx} = -1$

$(0,-2)$ $\frac{dy}{dx} = -\frac{1}{2}$

5. Consider the differential equation $\boxed{\frac{dy}{dx} = \frac{x+1}{y}}$

a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated and for -1< x < 1, sketch the solution curve passing through the point (0, -1).



$y = 0$ $\frac{dy}{dx} = \text{und}$

$x = -1$ $\frac{dy}{dx} = 0$

b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane for which $y \neq 0$. Describe all points in the xy-plane, $y \neq 0$, for which $\frac{dy}{dx} = -1$

c) Find the particular solution y = f(x) to the given differential equation with the initial condition f(0) = -2.